Contents

Welcome to the Rotman School of Management (Executive MBA Programs) math helpguide.

Our Executive MBA Diagnostic Tool (or EDT for short) has been introduced as a replacement for the GMAT for candidates who have an undergraduate degree recognized by the University of Toronto. Given the large scope of the testing in the GMAT, preparation time and anxiety levels were high for most candidates in the past. Also, many of the topics tested in the GMAT were simply not relevant for an Executive MBA Program.

The EDT is a targeted tool, developed by the Rotman faculty, which focuses on the relevant quantitative areas for your curriculum and requires little, or certainly much less, preparation time than the GMAT.

A typical EDT has eight to ten quantitative questions, a graph of a function, two or three logical reasoning questions, and will ask you to write a short memorandum from data on a given topic (this memo should be written in full sentences and organized into three sections: an introductory paragraph; a body containing observations in the data; and a concluding paragraph containing your insights into the topic). The purpose of the EDT is to get you ready for the Executive MBA Program, for those first days in the classroom and moments where quantitative discussion may occur. We believe that after passing the EDT, you will be well prepared to embark on your MBA adventure.

In this guide, we review a number of subjects that are frequently tested on the Rotman EDT: arithmetic, algebra, exponents and radicals, co-ordinate geometry, sequences and summations, and logarithms. We also talk about translating word problems into algebra. This guide is meant to be used to refresh your memory on topics that you may not have looked at for a while (it might be best not to think about exactly how long it has been since grade 10 math!) and not as a means to approach these topics for the first time. It is also important to note that just reading these materials is unlikely to be sufficient to prepare you for the test. You should also practice what you learn until you are completely comfortable with the subject matter.

Each chapter of the guide begins with a quiz. If you can answer all the quiz questions correctly, then you may not need to review that particular chapter. If you struggle with any of the questions (remember, on the Rotman EDT you need to show all your work to get full marks), then you should at least review the parts of the chapter relevant to the questions that gave you some trouble. Even if you get all the questions right, you may still want to do a quick review. In order to schedule your EDT exam, please contact 416-946-3022.

Enough introduction — on to the math!

Chapter 1

Arithmetic

Pre-Quiz

1. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=$
2. $\left(\frac{3}{5}-\frac{2}{3}\right) \div \left(\frac{1}{4}\times\frac{8}{3}\right) =$
3. Express the following as a decimal:
   a. $\frac{3}{4}$
   b. $\frac{9}{10}$
   c. $\frac{7}{5}$
   d. $\frac{5}{8}$
   e. $\frac{13}{2}$
   f. $\frac{5}{6}$
4. Solve the following percent problems:
   a. What is 50% of 80?
   b. What is 80% of 50?
   c. What is 40% of 90?
   d. What is 250% of 50?
   e. What is 250% more than 50?
   f. What is 25% more than 200?
   g. What is 35% more than 75% of 160?
   h. What is 40% less than 20?
   i. What is 60% less than 60% more than 100?

Fractions, Percents and Decimals

Fractions, percents and decimals are all different ways of expressing the same idea, and it is important to be able to move freely among these concepts.

SECTION 1 – FRACTIONS

A fraction is the relationship between a part and a whole. We call the top of the fraction the numerator and the bottom of the fraction the denominator. Expressed simply, a fraction is $\frac{part}{whole}$.

For example, the fraction $\frac{2}{3}$ means 2 parts out of a total of 3.

Mathematically, we can solve fractions by dividing the numerator by the denominator. In other words, $\frac{2}{3}$ can also be expressed as $2 \div 3$.

It is stylistically correct to reduce fractions to the lowest numbers possible. To reduce a fraction, look for a common factor of both the numerator and denominator and divide through by that factor. For example, $\frac{2}{4}$ (common factor of 2) can be reduced to $\frac{1}{2}$; $\frac{12}{16}$ (common factor of 4) can be reduced to $\frac{3}{4}$; $\frac{6}{9}$ (common factor of 3) can be reduced to $\frac{2}{3}$. When presenting the answer to a question, always reduce fractions as much as possible.

A fraction greater than 1, such as $\frac{7}{4}$ (where the numerator is greater than the denominator) is called an improper fraction. Improper fractions can be expressed as a mixed fraction where the whole is accompanied by a remainder. Since 4 divides into 7 once and leaves a remainder of 3, $\frac{7}{4}$ can be expressed as $1\frac{3}{4}$. Both forms – improper and mixed – are appropriate forms of expressing an answer, though it is often easier to work with improper fractions when performing basic mathematical operations.

You may be asked to perform any of the basic mathematical operations – addition, subtraction, multiplication and division – on fractions. It is important to be comfortable with all of these operations, so let's take a look at how each one works.

Multiplication

Multiplication is the easiest operation to perform on fractions. To multiply a series of fractions, one multiplies all of the numerators and then all of the denominators.

In the abstract, we get $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

For example:

$$\frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} = \frac{1\times2\times3}{4\times5\times7} = \frac{6}{140}$$

$$\frac{3}{4} \times \frac{2}{3} \times \frac{5}{6} = \frac{3\times2\times5}{4\times3\times6} = \frac{30}{72}$$

In both of these examples, we can reduce the answer (and should, to be mathematically correct). In the first case, $\frac{6}{140}$, both the numerator and the denominator are multiples of 2, so we can reduce our answer to $\frac{3}{70}$. In the second case, both the numerator and the denominator are multiples of 6, so we can reduce our answer to $\frac{5}{12}$.

Another useful thing to remember is that it does not matter in which order we multiply a set of numbers. Let's see how that rule can help us simplify fractions.

Here is our question:

What is the product of the fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5},$ and $\frac{5}{6}$?

Well, one way we could approach the question is to write it out as one big product in the order given: $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$. We could then multiply $\frac{1\times2\times3\times4\times5}{2\times3\times4\times5\times6} = \frac{120}{720}$. Since 120 goes into 720, we're left with $\frac{1}{6}$.

However, reordering the numerators and denominators gives us the following product: $\frac{2\times3\times4\times5\times1}{2\times3\times4\times5\times6}$. As we can see, $2 \times 3 \times 4 \times 5$ is present in both the numerator and the denominator and, accordingly, cancels out. This leaves us with $\frac{1}{6}$, the same answer using far less work!

Division

Many mathematical concepts are intertwined. One example is multiplication and division, which are opposite ways of expressing the same idea.

As a result, an alternative to dividing by a number is to multiply by the number's reciprocal. The reciprocal of a number is $\frac{1}{the\ number}$ or, more simply put, the number flipped upside down. For integers, we have to remember there is a hidden "1" in the denominator (for example, 2 is really $\frac{2}{1}$). So, the reciprocal of any integer *x* is $\frac{1}{x}$.

For example, the reciprocal of 3 is $\frac{1}{3}$; the reciprocal of 5 is $\frac{1}{5}$; the reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$; (or simply 3); and the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

So, when we divide by fractions, we invert the fraction in the denominator and then multiply.

In the abstract, we get $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$.

Here are some examples:

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{5}{3} \div \frac{2}{9} = \frac{5}{3} \times \frac{9}{2} = \frac{45}{6} = \frac{15}{2}$$

$$3 \div \frac{1}{2} = \frac{3}{1} \times \frac{2}{1} = \frac{6}{1} = 6$$

Addition and Subtraction

Before we can add or subtract fractions, we need to give them a common denominator.

For example, there is no way to add $\frac{1}{2}$ and $\frac{1}{3}$ in their current forms. Before we can do so, we need to convert each fraction so that it has the same number on the bottom as the other.

The simplest way to find the common denominator of two fractions is to multiply the denominators together. In our example, the denominators are 2 and

3. We can find a common denominator by multiplying 2 × 3, giving us 6.

However, our work is not done. If we are changing the denominator of each fraction, we need to also change the numerators in the same way (otherwise, we will be changing the value of each fraction). Since we multiplied the denominators together, we need to cross-multiply the numerator and denominator of each fraction to keep their values constant.

Here is the general rule using variables: if we are adding $\frac{a}{b}$ and $\frac{c}{d}$, we are going to end up with up with $\frac{ad+cb}{bd}$; or, if we are subtracting, we are going to end up with $\frac{ad-cb}{bd}$.

Let's look at a couple of examples to make things more concrete.

$\frac{2}{5}+\frac{3}{4}$: our common denominator is 5 × 4 = 20. For the top, we need to cross-multiply the numerators and denominators, which gives us 2 × 4 and 3 × 5. Since it is an addition problem, we add the results.

So: $\frac{(2\times4)+(3\times5)}{5\times4}=\frac{8+15}{20}=\frac{23}{20}$.

$\frac{5}{6}-\frac{2}{7}$: our common denominator is 6 × 7 = 42. For the top, we need to cross-multiply the numerators and denominators, which gives us 5 × 7 and 2 × 6. Since it is an subtraction problem, we subtract the results.

So: $\frac{(5\times7)+(2\times6)}{6\times7}=\frac{35-12}{42}=\frac{23}{42}$.

SECTION 2 – PERCENTS

Percents are very similar to fractions, with one difference - we're multiplying the end result by 100.

Here is the general formula for percents: percentage = $\frac{part}{whole} \times 100$.

Here are some examples:

What is 25% of 40? We are being asked to solve for the part, so let's rearrange the general equation to get *part* = *percentage* × *whole*. If we call the part *x*, we get *x* = 25% (40). On a calculator we can just plug in the numbers; without a calculator, we want to convert 25% to either a decimal or a fraction. Since we have been working with fractions so far, let's change 25% to $\frac{1}{4}$ (knowing the common fraction:decimal:percent equivalencies can make your life easier – and they are provided a bit later in this lesson). Then multiplying $\frac{1}{4} \times 40$, gives us our result of 10.

36 is what percent of 60? Here we are asked to solve for the percent, so we do not need to rearrange the equation. If we let the percent be *x*, we get $x = \frac{36}{60} \times 100\%$. So, $x = \frac{3}{5} \times 100\%$. We remember that we can write 100% as $\frac{100\%}{1}$, so when we multiply we get $\frac{3 \times 100\%}{5 \times 1} = \frac{300\%}{5} = 60\%$.

SECTION 3 – DECIMALS

Decimals are perhaps the easiest form of numbers to work with if you have a calculator at your disposal. The decimal form of a fraction is what you get after you actually divide the numerator by the denominator. For example:

$$\frac{2}{5} = 2 \div 5 = 0.4$$

$$\frac{3}{8} = 3 \div 8 = 0.375$$

As promised, here are some common fraction:decimal:percent equivalencies.

$\frac{1}{2} = 0.5 = 50\%$

$\frac{1}{3} = 0.333 = 33.3\%$

$\frac{1}{4} = 0.25 = 25\%$

$\frac{1}{5} = 0.2 = 20\%$

$\frac{1}{6} = 0.167 = 16.7\%$

$\frac{1}{7} = 0.142 = 14.2\%$

$\frac{1}{8} = 0.125 = 12.5\%$

$\frac{1}{9} = 0.111 = 11.1\%$

$\frac{1}{10} = 0.1 = 10\%$

$\frac{1}{20} = 0.05 = 5\%$

Chapter 2

Exponents and Radicals

Pre-Quiz

1. $2^4 =$

2. If $x^2 = 36$, solve for x.

3. Simplify each of the following (i.e. do not calculate the answer, just combine the components to form a new term – the first one is solved as an example):

a. $2^3 \times 2^4 = 2^7$

b. $3^5 \times 3^4$

c. $7^5 \div 7^3$

d. $(2^4)^3$

e. $2^4 + 2^5$

4. $\sqrt{25} =$

5. $\sqrt{36} + \sqrt{49} =$

6. Simplify Each of the following:

a. $\sqrt{75} \div \sqrt{3}$

b. $\sqrt{240} \div \sqrt{12}$

c. $\sqrt{18} \times \sqrt{8}$

d. $(\sqrt{75})^2$

e. $(25)^{\frac{1}{2}}$

f. $\sqrt[3]{125}$

Exponents and radicals (also known as "roots") are most frequently seen in statistical analyses. It is important to understand how they work to be able to understand statistical data. For the purposes of the Rotman aptitude test, you will need to know how to simplify expressions containing exponents and roots and to solve quadratic equations (which we will learn about in the algebra section).

SECTION 1 – EXPONENTS

The most common exponent you will see on the exam is the square (or power of 2). One key fact to remember about squares is that they have two roots - one positive and one negative.

For example, if we know that $x^2 = 25$, $x = 5$ and $x = -5$ are solutions.

There are certain rules to remember when performing the basic operations on exponents:

$$x^a \times x^b = x^{a+b}$$

$$(x^a)^b = x^{a+b}$$

$$x^a \div x^b = x^{a-b}$$

Here are some concrete examples:

$$2^5 \times 2^4 = 2^{5+4} = 2^9$$

$$(2^5)^4 = 2^{5\times4} = 2^{20}$$

$$2^5 \div 2^4 = 2^{5-4} = 2^1 = 2$$

Also, it is important to recognize that there is no easy way to add or subtract exponents. So:

$x^a + x^b$ **does not** $=$ x^{a+b}; and

$x^a - x^b$ **does not** $=$ x^{a-b}

SECTION 2 – RADICALS

Radicals are the counterpoint to exponents; just as the square of 5 is 25, the root of 25 is 5 (or possibly −5, if you remember the discussion above).

The radical sign, $\sqrt{\ }$, means "the positive root of". So, if you see $x = \sqrt{25}$ on the test, you know you only have to worry about the positive solution; in other words, *x* = +5. Just like for exponents, there are some basic rules to remember when simplifying or solving for radicals:

$$\sqrt{x} \times \sqrt{y} = \sqrt{xy}$$

$$\sqrt{x} \div \sqrt{y} = \sqrt{\frac{x}{y}}$$

Here are some concrete examples:

$$\sqrt{12} \times \sqrt{3} = \sqrt{36} = 6$$

$$\sqrt{54} \div \sqrt{6} = \sqrt{9} = 3$$

Sometimes we are asked to simplify a root that is not a perfect square. To do so, we use one of our two basic rules. For instance:

Simplify $\sqrt{54}$.Well, 54 is not a perfect square. Sure, we could use our calculator, but that is going to give us a precise value, not what the question is asking for with the term "simplify" (if the test maker wanted a precise value, they would have asked us to "solve" instead of simplify).We can only simplify if we have a perfect square involved, so somehow we need to get one in our expression.

From our first rule of radicals, we know that we can rewrite $\sqrt{xy}$ as $\sqrt{x} \times \sqrt{y}$. To simplify a radical, we want to break it up into the product of two radicals, one of which is a perfect square. So, we ask ourselves, what perfect square is a factor of 54? The number 9 is both a perfect square and a factor of 54, so we can rewrite $\sqrt{54}$ as $\sqrt{9} \times \sqrt{6} = 3\sqrt{6}$, our final answer.

Simplify $\sqrt{75}$. Using the same method as above, $\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

Simplify $\frac{\sqrt{180}}{\sqrt{30}}$. Using our division rule, we know that $\frac{\sqrt{180}}{\sqrt{30}} = \sqrt{\frac{180}{30}} = \sqrt{6}$

Similar to exponents, there is no easy way to add or subtract radicals. So:

$\sqrt{x} + \sqrt{y}$ **does not** $=$ $\sqrt{x+y}$; and

$\sqrt{x} - \sqrt{y}$ **does not** $=$ $\sqrt{x-y}$

Chapter 3

Algebra

Pre-Quiz

1. If $2x + 5 = 35$, what is the value of x?

2. If $-3y - 12 = -2y + 6$, what is the value of y?

3. Isolate x in each of the following equations:
   a. $2y - 3x = 15$
   b. $\frac{8x+10y}{2x} = 6$
   c. $2x + 4y = 3z - 8 + 5x$
4. If $x - 2y = 10$ and $3x + 2y = 14$, what are the values of x and y?

5. If $2x + 3y = 12$ and $5x - 6y = 39$, what are the values of x and y?

6. If $x^2 + 4x - 12 = 0$, what are the possible values of x?

7. If $y^2 - 12y + 20 = 0$, what are the possible values of y?

8. Expand each of the following (the first is done as an example):
   a. $(z + 12)(z - 4) = z^2 + 8z - 48$
   b. $(c + 4)(c - 4)$
   c. $(2x + 3)(3x - 4)$
   d. $(x + 4)(y - 2)$
   e. $(x^2 - x)(x + 4)$

Algebra is one of the most pervasive topics in all of math. Whether we are simplifying an expression, solving an equation or inequality or digging our way through a complicated word problem, algebra is involved. In this chapter we will start with the basics and work our way up to the interesting stuff.

Manipulating Equations

There is one big rule to remember when solving an equation: you can do pretty much anything you want, but whatever you do to one side of the equation you also have to do to the other side.

Let's start with a simple example:

If $x + 10 = 25$, what is the value of x?

If we are solving for a variable, our job is to isolate that variable - in other words, get it by itself on one side of the equation. In our example, x is accompanied by 10. To isolate x, we need to perform an operation that is the opposite of the current relationship between x and 10. Here, 10 is being added to x. To remove 10 from the left side of the equation, we are going to need to subtract it from both sides to maintain the equivalence between both sides of the equation.

$x + 10 = 25$

$x + 10 - 10 = 25 - 10$ (note that we subtracted 10 from both sides)

$x = 15$

Mentally, you might think of the process as "moving the 10 over to the other side" — and that is a perfectly acceptable way to visualize what you are doing. In more complicated scenarios; however, making sure you repeat the operation on both sides of the equation will vastly reduce the chance for error.

Let's look at some more examples:

If $10x - 5 = 6x + 15$, what is the value of x?

Our first step is to move every x to one side of the equation. Accordingly:

$10x - 5 - 6x = 6x + 15 - 6x$

$4x - 5 = 15$

Next, let's get rid of that −5.

$4x - 5 + 5 = 15 + 5$

$4x = 20$

Finally, since x is currently being multiplied by 4, we need to do the opposite and divide both sides by 4.

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

So far, we have only been looking at equations with one variable. Adding a second (or third) variable might make things a bit trickier, but it will not change the basic rules of the game.

If $3x + 4y = 15$, what is the value of x?

$3x + 4y - 4y = 15 - 4y$

$3x = 15 - 4y$

$x = \frac{15-4y}{3}$

$x = 5 - \frac{4y}{3}$

If $\frac{2y+6}{3x} = 5y$, what is the value of *x*?

$\frac{2y+6}{3x} \times 3x = 5y \times 3x$

(we always want to get our desired variable out of the denominator - to do so here we need to multiply both sides by 3*x*)

2*y* + 6 = 15*xy* (now we want to isolate the *x* from the15x*y*, so let's divide both sides by 15*y*).

$\frac{2y+6}{15y} = \frac{15xy}{15y}$

$\frac{2y}{15y} + \frac{6}{15y} = x$

$x = \frac{2}{15} + \frac{2}{5y}$

Solving Systems of Equations

So far we have seen what happens when there is only one equation involved. In more complicated problems; however, you are likely to see multiple equations. Here is the big rule to remember:

<u>To fully solve a system containing *n* variables, one requires *n* distinct linear equations.</u>

For example, if we are being asked to solve for the values of variables *x*, *y* and *z*, we are going to need three distinct linear equations.

"Distinct" just means "different". For example, the equations *x* + *y* = 10 and *x* + 2*y* = 25 are different; however, the equations *x* + *y* = 10 and 2*x* + 2*y* = 20 are not, as the second is just a multiple of the first.

"Linear" means "contains no exponent greater than 1". *x* + *y* = 10 is a linear equation; x^2 = 25 is not.

There are two different methods of solving systems of equations: substitution and combination. Each one is equally accurate, but in some situations, one may be quicker than the other.

Substitution

In school, students learn substitution. In this method, we isolate the variable that we want to eliminate (or, if we are solving for all the variables, the one that is easier to isolate), then substitute that value back into the other equation. Here is an example:

If $x + 4y = 10$ and $x - 2y = -2$, what is the value of y?

We want to solve for y; however, let's start by isolating x. We subtract $4y$ from both sides of the first equation and get:

$$x + 4y - 4y = 10 - 4y$$

$$x = 10 - 4y$$

Now, let's plug in $(10 - 4y)$ for x in our second equation. Accordingly, we get:

$$(10 - 4y) - 2y = -2$$

$$10 - 6y = -2$$

$$10 - 6y + 6y = -2 + 6y$$

$$10 = -2 + 6y$$

$$10 + 2 = -2 + 6y + 2$$

$$12 = 6y$$

$$\frac{12}{6} = \frac{6y}{6}$$

$2 = y$... voilà!

Combination

Combination is an often-overlooked method for solving systems of equations. Combination relies on our ability to add and subtract equations to form new ones. When equations line up favourably, combination can be much quicker than substitution.

Let's revisit the previous question:

If $x + 4y = 10$ and $x - 2y = -2$, what is the value of y?

Again, we want to solve for y, so we want to make x disappear. This time, however, we notice that each equation has the same number of x's. As a result, if we subtract the second equation from the first, x will completely vanish from the question.

Let's line up the equations, one on top of the other. To keep things positive, we will put the equation with the greater number of y's on top:

$x + 4y = 10$

$x - 2y = -2$

Now, we subtract the entire second equation from the first one:

$x - x = 0x$ (or nothing)

$4y - (-2y) = 6y$

$10 - (-2) = 12$

So, we now have:

$0x + 6y = 12$ (or just $6y = 12$)

$$\frac{6y}{6} = \frac{12}{6}$$

$y = 2$

As you can see, when combination works, it requires a lot less time and effort than substitution (yet it's equally, if not more, elegant).

System of Equations Word Problems

In some cases, we are not given equations containing variables set up for us to solve, rather we are given information relating two or more objects in sentence form. When presented with a word problem, you first need to define the relevant variables, then create the equations that reflect the information given in the statement. Once the equations have been set up you can use combination or substitution to solve for your unknowns.

Let's look at an example.

Arun is three times as old as Becca. Eight years from now, Arun will be twice as old as Becca. How old is Becca now?

First, we define variables to represent the ages of Arun and Becca now.

A = Arun's age now
B = Becca's age now

Next, we set up our equations. Remember that we need as many equations as we have variables – in this case two.

Our first piece of information tells us the relationship between Arun and Becca's ages now. So,

$A = 3B$
(in other words, the value of Arun's age now will be Becca's age now times 3)

The next piece of information relates their ages eight years in the future, so the relationship will apply to their current ages increased by 8.

$(A + 8) = 2(B + 8)$

We can then solve our two equations using either of the methods introduced above – substitution will work well here by substituting $3B$ for A in our second equation.

$((3B) + 8) = 2(B + 8)$

$3B + 8 = 2B + 16$

$3B - 2B = 16 - 8$

$B = 8$

Therefore, Becca is 8 years old now.

Quadratic Equations

Quadratic equations, sometimes known as binomials, are those which contain a squared term. For example, $x^2 + 7x + 10 = 0$ or $b^2 - 5b - 24 = 0$.

In general, you may be asked to do one of two things with a quadratic equation: expand it or factor it out (and solve it). The two examples above are in expanded form. In factored form, they would appear as $(x + 5)(x + 2) = 0$ and $(b - 8)(b + 3) = 0$, respectively.

To expand a quadratic, we use the method you probably remember from high school - FOIL. FOIL stands for "first, outer, inner, last", referring to how one can methodically multiply each part of the quadratic. Let's do an example using the FOIL method.

Expand $(z + 10)(z - 5)$

F means we multiply the first term of each bracket: $z \times z = z^2$

O means we multiply the outer term of each bracket: $z \times (-5) = -5z$

I means we multiply the inner term of each bracket: $10 \times z = 10z$

L means we multiply the last term of each bracket: $10 \times (-5) = -50$

Put it all together and we get $z^2 - 5z + 10z - 50$. Combine like terms and we finally get $z^2 + 5z - 50$.

To factor a quadratic, we use reverse FOIL. Going backwards can be a bit more complicated, as there are a few extra steps to remember.

Factor $z^2 + 5z - 50$

The first thing we look at is the coefficient (i.e. the number) in front of the squared term. The vast majority of the time it is just going to be a "silent 1" (in other words, no visible number). If that is the case, we can start each bracket with the variable. So, thus far we have:

$(z \quad)(z \quad)$

Next, we look at the far-right sign (i.e. the one before the last term).
If that sign is positive, we know that we need to multiply either two positive numbers or two negative ones. If that sign is negative, we know that we need to multiply a positive by a negative. In our example, the last term is −50; the only way we can get a negative product is to mix the signs in the brackets. So:

$(z + \quad)(z - \quad)$

Now we need to figure out what our numbers are. The final term is 50, so we need two numbers that multiply to 50.The middle term is +5z, so we need two numbers that are 5 apart (since one is positive and one is negative – if both numbers had had the same sign, we would have looked for two numbers that add up to 5). So, which two numbers multiply to 50 and subtract to 5? 10 and 5!

Finally, we need to figure out which is positive and which is negative; since we ended up with +5z, the bigger number has to be positive. Our final solution:

$(z + 10)(z - 5)$

Let’s try one more example (without all the verbiage):

Factor $x^2 - 8x + 12$

Step 1: $(x \quad)(x \quad)$

Step 2: Second sign is positive, so both brackets will contain the same sign; we ended up with $-8x$, so both signs must be negative.

$(x - \quad)(x - \quad)$

Step 3: Find two numbers that multiply to 12 and add up to 8: 6 and 2!

$(x - 6)(x - 2)$

Note: since both signs are the same, it doesn’t matter which number we put in which bracket.

Let’s finish up this section by going one step further - actually solving a quadratic equation.

If $x^2 - 8x + 12 = 0$, what are the two possible values of x?

We picked the same example as above to speed up the process a bit. Using our previous work, we can factor out the quadratic and get our new equation:

$(x - 6)(x - 2) = 0$

Now let’s think about a basic principle of multiplication: the only way to get a product of 0 is to multiply by 0. So, if we know that $xy = 0$, we also know that either $x = 0$ or $y = 0$.

The same principle applies to quadratics. If $(x - 6)(x - 2) = 0$, then either $(x - 6) = 0$ or $(x - 2) = 0$. Accordingly, either $x = 6$ or $x = 2$.

So, if you are asked to solve a quadratic equation, always start by setting the right side of the equation to 0 (you might have to manipulate the equation to do so), then follow the steps outlined above.

Quadratic Formula by Completing the Square

You will have noticed that with a quadratic equation it may be difficult to isolate x using basic operations on account of the x^2 term. Isolating the x term is possible using a more advanced mathematical method called *completing the square*. For the purposes of the EDT this method is likely unnecessarily complex and solving by factoring as shown above should be sufficient and we will not review the details of the method here. However, there is a very useful result we can use when we apply completing the square to a generic form of a quadratic equation as given by,

$$ax^2 + bx + c = 0$$

When we isolate x in this generic equation, we arrive at a formula that allows us to calculate x based on the coefficients of the three terms in our quadratic equation – a, b and c. This formula is called the *quadratic formula* and is given as,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sign of a Quadratic Function

When solving a quadratic equation – by factoring or using the quadratic formula – you are finding the values of x where the function is equal to zero (hence setting the right side of the equation equal to zero). We can extend this concept to be able to characterize if a quadratic function is always positive, always negative, or sometimes positive and sometimes negative depending on the value for x.

If we wanted to know the sign of the quadratic function, $f(x) = x^2 - 8x + 12$, finding the values of x at which the function is zero ($x = 6$ and $x = 2$) tells us that the value of the function changes from positive to negative as it passes through one zero, and then changes from negative to positive as it passes through the second zero.

Also, one can test the function with different values for x to see how the function behaves.

For example, a value of x between the values of 2 and 6, such as 4, will yield a negative result.

$f(4) = (4)^2 – 8(4) + 12 = 16 – 32 + 12 = – 4$

Whereas a value of x outside the values of 2 and 6, such as 1, will yield a positive result.

$f(1) = (1)^2 – 8(1) + 12 = 1 – 8 + 12 = 5$

So, the function $f(x)$ is positive or negative depending on the value of x.

The sign of a quadratic function and the way it changes or does not change will be further illustrated when we consider the graphical representation of these functions in the chapter on co-ordinate geometry.

Absolute Values

An absolute value looks at the value of a number or algebraic expression irrespective of the sign (i.e. positive or negative). In other words,

$|5| = 5$, and

$|-5| = 5$

Another way to conceptualize the absolute value is to consider the distance of both values from zero. Both 5 and –5 are the same distance from zero and have equivalent absolute values.

When absolute values are found in algebra equations, they require special care as two cases result. For example,

$|x + 1| = 2x - 3$

Similar to our simple example above, we can see the expression contained within the absolute value brackets can be equal to $(2x – 3)$, or $–(2x – 3)$. Therefore, we need to consider both cases:

$x + 1 = 2x - 3$, and

$$x + 1 = -(2x - 3)$$

Solving these two equations gives $x = 4$, and $x = \frac{2}{3}$ respectively.

However, once we solve these two cases, we need to verify if the solutions satisfy the original equation. Taking our first solution $x = 4$ yields the following:

$$|(4) + 1| = 2(4) - 3$$

$$|5| = 8 - 3$$

$$|5| = 5$$

Since this is a true statement, we know that $x = 4$ is a valid solution to our equation.

Taking our second solution of $x = \frac{2}{3}$ yields the following:

$$\left|\left(\frac{2}{3}\right) + 1\right| = 2\left(\frac{2}{3}\right) - 3$$

$$\left|\frac{2}{3} + \frac{3}{3}\right| = \frac{4}{3} - \frac{9}{3}$$

$$\left|\frac{5}{3}\right| = -\frac{5}{3}$$

Since this statement is not true, we know that $x = \frac{2}{3}$ is not a valid solution to our equation. Absolute value equations can have one, two, or possibly even zero solutions.

Inequalities

Just as in life, not everything in math is equal. Sometimes, instead of solving an equation, we are asked to solve an inequality. Unlike equations, which have specific numbers as solutions, inequalities have a range of numbers as solutions.

Here is an example:

If $z + 14 > -5$, what is the possible range of values for z?

For the most part, we treat inequalities exactly the same as equations (with one very important exception, which we'll discuss below). So, since we are being asked to solve for z, we need to isolate it. This example is pretty simple.

$z + 14 - 14 > -5 - 14$ (remembering to do the same thing to both sides)

$z > -19$

The big exception to treating inequalities identically to equations occurs when we multiply or divide both sides of the inequality by a negative number. When that happens, we need to remember to *swap the inequality*. For example:

If $-3x - 10 > 11$, what is the possible range of values for x?

We start the same way as always – isolate x. So:

$-3x - 10 + 10 > 11 + 10$

$-3x > 21$

In our next step; however, we are going to divide both sides by −3. We must change the direction of our inequality.

$\frac{-3x}{-3} > \frac{21}{-3} \rightarrow x < -7$

It is important to remember this key difference between inequalities and equations.

Chapter 4

Co-ordinate Geometry

Pre-Quiz

1. What are the y-intercept, x-intercept and slope of the following lines?

   a. $y = 2x - 10$

   b. $y = 9x + 18$

   c. $x = \frac{1}{2}y - 9$

   d. $2y + 3x = 16$

2. What is the equation of the line with a slope of $-\frac{1}{2}$ that passes through the point (2,4)?

3. Find the equations of the lines that pass through each of the following pairs of points:

   a. (–2, 1) and (4, –1)

   b. (2, 2) and(4, 8)

   c. (3, 4) and (7, 8)

   d. (0, 2) and (–4, 14)

4. What is the distance between the points (–8, –4) and (–4, –1)?

5. Graph the following:

   a. $y = 4x - 2$

   b. $4y = 2x - 16$

   c. $x = \frac{1}{2}y + 4$

   d. $y = x^2 - 2x - 3$

Co-ordinate geometry involves mapping points, lines and functions in an *x-y* plane. For the Rotman aptitude test, you need to understand how to find the equation of a line and how to graph linear and quadratic functions.

The *x-y* Plane

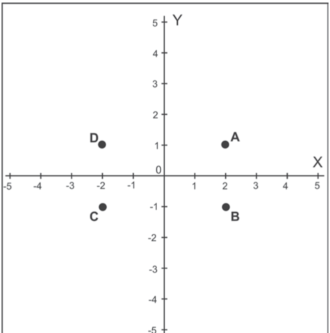


Figure 4.1

The figure above represents the standard *x-y* co-ordinate plane. Each point on the plane is described in the form (*X*, *Y*), where *X* is the horizontal co-ordinate and *Y* is the vertical co-ordinate. When assigning values to the points, "right" and "up" are considered positive and "left" and "down" are considered negative.

For example, point *A* above is two points to the right of (0, 0) (also known as the origin) and one point above (0, 0), so we would call point *A* (2, 1). Along the same lines:

Point *B* is two points to the right of the origin and one point down, so we would call point *B* (2, −1).

Point *C* is two points to the left of the origin and one point down, so we would call point *C* (−2, −1).

Point *D* is two points to the left of the origin and one point up, so we would call point *D* (−2, 1).

Lines on the Co-ordinate Plane

The equation of a line on the x-y plane is commonly written in the form

$y = mx + b$, where

m = the slope of the line; and

b = the y-intercept of the line.

To find the equation of a line, one needs either two points on the line or one point on the line and the slope.

Slope

Slope is sometimes referred to as “the rise over the run”. In other words, the slope measures how steep the line is. The slope of a line is,

$$\frac{\textit{the change in vertical distance}}{\textit{the change in horizontal distance}}$$

Mathematically speaking, if we want to calculate the slope of a line that contains the points (1, 3) and (4, 15), we would divide the change in y by the change in x.

So:

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{3-15}{1-4} = \frac{-12}{-3} = 4$ Therefore, the slope of this line is 4.

Let’s examine one more example:

What is the slope of the line passing through points (4, 2) and (−2, 4)?

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{2-4}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$$

The *y*-intercept

The *y*-intercept of a line is the value of *y* where the line crosses the *y*-axis; in other words, the value of y when $x = 0$.

To solve for the *y*-intercept of a line, we need the slope of the line and at least one point. To illustrate, let's go back to two points for which we have already determined the slope, (1, 3) and (4, 15).

From our previous work, we know that the slope of the line containing these two points is 4. So, we can plug 4 into our generic equation for a line and get:

$y = 4x + b$ (remember, *b* is our *y*-intercept)

Now, to solve for *b*, we need values for *x* and *y*. Since we know that the point (1, 3) is on the line, we can plug in $x = 1$ and $y = 3$. Now, our equation reads:

$3 = 4(1) + b$; or simply

$3 = 4 + b$

Going back to our basic rules of algebra, we can solve for b as follows: 3

$- 4 = 4 + b - 4$

$-1 = b$ (or, if you prefer, $b = -1$)

Now that we have the value of *b*, we can write our equation of the line in its final form:

$y = 4x - 1$

Plotting Functions

In some questions you may be asked to graph a function. These functions can be linear or quadratic functions. As the name implies, plotting a linear function results in a line, whereas plotting a quadratic function results in a parabola. We will review each of these in turn starting with a line.

Graph $3 = 12x - 3y$.

To do so, the first step is to rearrange the equation that you are given into the standard $y = mx + b$ form. In other words, we want to solve for y. Using the above example:

$3 = 12x - 3y$

$3 + 3y = 12x - 3y + 3y$

$3 + 3y = 12x$

$3 + 3y - 3 = 12x - 3$

$3y = 12x - 3$

$\frac{3y}{3} = \frac{12x-3}{3}$

$y = 4x - 1$

Now that we have our equation, we start by plotting the y-intercept. We have a value for b of -1, so we know that the line crosses the x-axis at $(0, -1)$.

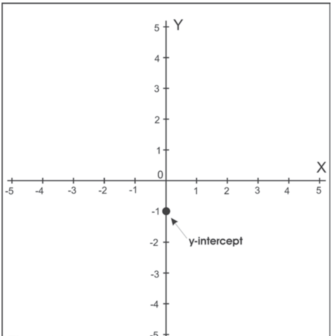


Figure 4.2

Next, we use the slope of the line to find another point. Since the value of m is 4, we know that the line goes up 4 points for every 1 point that it goes right. So, counting off the points on our diagram, we see that 1 to the right takes us to $x = 1$ and 4 up takes us to $y = 3$. Our next point is, therefore, (1, 3). Now we draw the line of infinite length (or as much of it as we can fit on the paper) that passes through our two points (0, −1) and (1, 3) and we are done.

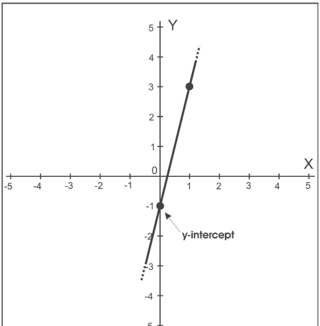
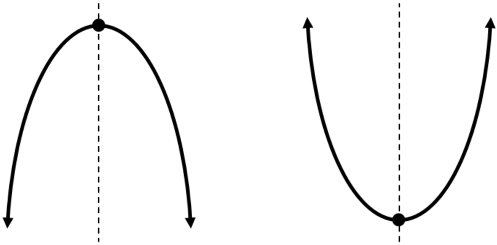


Figure 4.3

The second type of function we need to plot is a quadratic function. To do so, we first need to understand the characteristics of the shape of a quadratic plot – commonly referred to a s a parabola.

A parabola is a symmetric curve that is either oriented opening down and has a maximum value, or up and has a minimum value. The point at which we have this extreme value (maximum or minimum) is called the vertex and it sits on the axis of symmetry.

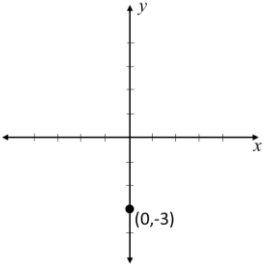


To plot a parabola, we need to identify the intercepts (both x and y), the location of the axis of symmetry and the vertex. Let's look at an example.

Graph $y = x^2 + 2x - 3$

The first point to identify is the y-intercept and can be calculated by solving for y when $x = 0$.

$y = (0)^2 + 2(0) - 3 = -3$ → The y-intercept is at (0, –3).

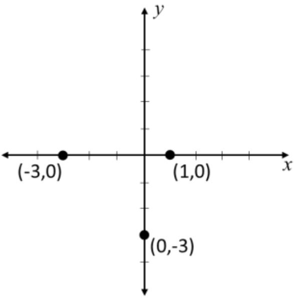


Next, we want to find the x-intercepts by solving for x when y equals zero. Notice that this step results in solving a quadratic equation as seen in Chapter 3.

$(0) = x^2 + 2x - 3$

$(x - 1)(x + 3) = 0$

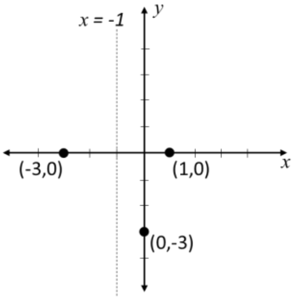
$x = 1$, and $x = -3$ → There are two x-intercepts at (1, 0) and (– 3, 0).



The final point we would like to plot is the vertex, which we know lies on the axis of symmetry. We can identify the x value for the axis of symmetry by identifying the midpoint between our two x-intercepts, or by using the following equation.

$x_{mid} = \frac{-b}{2a}$, where a and b are found in the quadratic $ax^2 + bx + c = 0$.

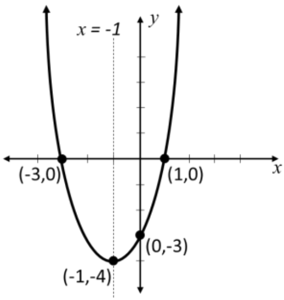
So, $x_{mid} = \frac{-(2)}{2(1)} = -\frac{2}{2} = -1$ → The axis of symmetry is at $x = -1$.



The y-coordinate for the vertex can be found by solving for y at the value of x_{mid}.

$y = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$ → Vertex is at $(-1, -4)$.

We complete plot of the parabola by plotting the vertex and drawing the curve.



Chapter 5

Sequences and Summations

Pre-Quiz

1. If S is the sequence of numbers defined by the equation $S_{n+1} = 2S_n - 4$ and if $S_1 = 6$, solve for the value of:
   a. S_2
   b. S_3
   c. S_4

2. If R is the sequence of numbers defined by the equation $R_{n+1} = nR_n - 7$ and if $R_4 = -16$, solve for the value of:
   a. R_3
   b. R_2
   c. R_1

3. Set Q is defined as the numbers generated by the rule "$Q_{n+1} = (Q_n - 3)^2 - 6$" for $1 \leq n \leq 4$. If $Q_1 = 6$, find the summation of set Q.

4. Given $\sum_{n=2}^{5} n^2 - 3n + 7$, write out the expression and evaluate it.

Sequences

A sequence is an ordered list of values defined by an equation. Most commonly, a sequence is defined by showing the relationship between the nth and $n+1$st terms.

For example:

$x_{n+1} = x_n + 3$, for all values of $n > 1$. In this sequence, each term is 3 more than the previous term.

$x_{n+1} = 3x_n - 5$, for all values of $n > 1$. In this sequence, each term is 3 times the previous term minus 5.

$x_{n+1} = 4x_n + 2$, for all values of $n > 1$. In this sequence, each term is 4 times the previous term plus 2.

To solve for a term in a sequence, one needs to know the defining rule and at least one term of the sequence; just knowing the rule is not enough, since we have no idea where the sequence begins. To illustrate this point, let's expand on two of our examples above:

$x_{n+1} = x_n + 3$, for all values of $n > 1$.

If $x_n = 5$, then the sequence is going to be the infinite set of numbers {5, 8, 11, 14, 17, ...}.

If, on the other hand, $x_n = 10$, then the sequence is going to be the infinite set of numbers {10, 13, 16, 19, 22, ...}.

Let's look at a test-like question:

In the infinite sequence B, $B_{n+1} = 2B_n - 3$, for all $n \geq 1$. If $B1 = 5$, what is the value of B_4?

We know the value of B_1, so we can plug $n = 1$ into our rule and get the following:

$B_2 = 2B_1 - 3$.

Replacing B_1 with 5, we get $B_2 = 2(5) - 3$ and, finally,

$B_2 = 7$

Of course, we are not done, since the question asks us to solve for B_4. We need to repeat the above steps twice more.

Now we know the value of B_2, so we can plug $n = 2$ into our rule and get:

$B_3 = 2B_2 - 3$

Replacing B_2 with 7, we get $B_3 = 2(7) - 3$ and, finally:

$B_3 = 11$

We now have the value of $B3$, so we can plug $n = 3$ into our rule and get:

$B_4 = 2B_3 - 3$

Replacing B_3 with 11, we get $B_4 = 2(11) - 3$ and, finally, our answer:

$B_4 = 19$.

Series

Series are the summations of finite (i.e. fixed number of terms) sequences. In other words, a series is the sum of all of the terms of a sequence.

Like many other abstract concepts, series are best understood by reviewing a concrete example.

1. The set Y is defined as the numbers generated by the rule $Y_n+1 = (Y_n)2 - 5$ for $2 \leq n \leq 5$. If $Y2 = 3$, what is the summation of set Y?

First, we need to solve for the terms in the sequence. We know that $Y_2 = 3$, so we can solve for Y_3 through Y_5 using the methods described earlier in this chapter.

$Y_3 = (Y_2)2 - 5$

$Y_3 = (3)2 - 5$

$Y_3 = 9 - 5 = 4$

$Y_4 = (Y_3)2 - 5$

$Y_4 = (4)2 - 5$

$Y_4 = 16 - 5 = 11$

$Y_5 = (Y_4)2 - 5$

$Y_5 = (11)2 - 5$

$Y_5 = 121 - 5 = 116$

So, our set is {3, 4, 11, 116} and our summation is:

3 + 4 + 11 + 116 = 134.

It is common to see summations introduced by the symbol $\sum$ with the first term listed on the bottom of the symbol and the final term listed on top. Here is an example:

Given $\sum_{n=3}^{6} n^2 + 4n - 5$, write out the expression and evaluate it.

To solve, we rewrite the expression for each value of n. So:

for $n = 3$, we get 9 + 12 – 5 = 16;

for $n = 4$, we get 16 + 16 – 5 = 27;

for $n = 5$, we get 25 + 20 – 5 = 40; and

for $n = 6$, we get 36 + 24 – 5 = 55.

We were asked for the summation, so we add our set of values to get:
16 + 27 + 40 + 55 = 138.

Chapter 6

Logarithms

Pre-Quiz

1. Solve for x in the following:
   a. $\log_4 16 = x$
   b. $\log_3 81 = x$
   c. $\log_x 25 = 2$
   d. $\log_x 32 = 5$
   e. $\log_6 x = 3$
   f. $\log_2 x = 6$
   g. $\log 100 = x$
   h. $\log x = 5$

2. Express each of the following as a single term:
   a. $\log_2 10 + \log_2(\frac{1}{2})$
   b. $\log 1000 + \log 100$
   c. $\log_3 108 - \log_3 4$
   d. $\log 1000 - \log 100$

Logarithms

Logarithms are very similar to exponents and the properties of logarithms can be expressed using the properties of exponents. While logarithms can get quite complex, for the Rotman aptitude test one only needs to be able to solve fairly basic logarithm questions.

Logarithms are usually expressed in the form:

$$\log_a M = x$$

This relationship is identical to the following:

$a^x = M$

In other words, a logarithm is just a different way of expressing an exponent. Here are some concrete examples to make this relationship clearer:

$\log_2 16 = 4$ ($2^4 = 16$)

$\log_3 9 = 2$ ($3^2 = 9$)

$\log_5 125 = 3$ ($5^3 = 125$)

If a logarithm is written with no subscripted number between the word "log" and the *M*, there is an implied base of 10. For example:

log100=2(log10100=2,since102=100)

log1000=3(log101000=3,since103=1000)

Let's try a couple of simple questions:

If $\log_5 N = 4$, what is the value of N?

Rewritingourequationinexponentform,weget: $5^4 = N$

$5 \times 5 \times 5 \times 5 = N$

$N = 625$

If $\log_9 81 = x$, what is the value of x?

Rewriting the equation in exponent form, we get: $9^x = 81$

$x = 2$

Logarithm Rules

Just as it is possible to combine exponential terms (under the right circumstances), it is also possible to combine logarithms. There are two basic rules that tell us when and how to combine logarithms.

1. *Product Rule for Logarithms*: the log of the product of two terms is equal to the sum of the individual logs. In other words,

   $\log_a MN = \log_a M + \log_a N.$

Here is an example of how we would apply this rule:

Solve for $\log_2(\frac{8}{3}) + \log_2(\frac{3}{2})$.

$\log_2(\frac{8}{3}) + \log_2(\frac{3}{2})$

$= \log_2(\frac{8}{3} \times \frac{3}{2})$

$= \log_2(\frac{24}{6})$

$= \log_2(4)$

If we let $\log_2 4 = x$, then $2^x = 4$. Therefore, $x = 2$.

2. *Quotient Rule for Logarithms*: the log of M divided by N is equal to the log of M minus the log of N. In other words,

   $\log_a(\frac{M}{N}) = \log_a M - \log_a N.$

Here is an example of how we would apply this rule:

Solve for $\log_5(1000) - \log_5(8)$.

$\log_5(1000) - \log_5(8)$

$= \log_5(\frac{1000}{8})$

$= \log_5(125)$

If we let $\log_5 125 = x$, then $5^x = 125$. Therefore, $x = 3$.

Chapter 7

Logic

Pre-Quiz

1. Translate each of the following into an *if-then* statement:
   a. All people in Toronto are in Ontario.
   b. Bob will win the race only if Sheila does not enter.
   c. Sam will get the promotion if Mary leaves the company.
   d. Yuko and Phillipe never go to the movies together.
   e. Sabrina and Juan never go to the movies apart.
   f. At least one of Hassan and Dietrich is always on vacation.
   g. Every time it rains or snows I get a headache.

2. Form the contrapositive of each of the above statements.

3. Kat is attending a sale at a local store. There are six items for sale: a bookcase, a chair, a desk, an encyclopedia, a fez and a grand piano. The following rules govern what she will buy:

   If Kat buys the desk, she also buys the chair.

   If Kat buys the encyclopedia, she also buys the bookcase.

   If Kat doesn't buy the fez, then she won't buy the grand piano. Kat won't

   buy both the chair and the bookcase.

i) If Kat buys the bookcase, which of the following must be true?
   a. She buys the fez.
   b. She buys the encyclopedia.
   c. She buys the chair.
   d. She doesn't buy the desk.
   e. She doesn't buy the grand piano.

ii) What is the maximum number of items that Kat can buy?
   a. 2
   b. 3
   c. 4
   d. 5
   e. 6

On the Rotman aptitude test, you will be asked to solve a logic puzzle. While it is often possible to solve these types of puzzles intuitively, knowledge of the basics of formal logic can make solving them much simpler. In this chapter, we will review translating formal logic statements and combining such statements.

Translating Formal Logic Statements

The basic unit of formal logic is the *if-then* statement. Not surprisingly, these statements have two parts: the *if* clause (also known as the trigger) and the *then* clause (also known as the result).

For example: If that animal is a dog, then it is a mammal.

The above statement might seem straightforward and intuitively easy to understand. In general, when learning new concepts, the best way to proceed is to start with something that makes sense and then work your way up to more complicated scenarios.

So, let's take apart our example and see exactly what we are saying from a logical perspective.

A trigger is a fact that guarantees a certain result. Whenever you see a trigger in a logic puzzle, you know with certainty that if the trigger happens, the result will happen. In other words:

Trigger ➔ Result

Based on our statement, if we know that something is a dog, it is guaranteed to be a mammal as well.

There are many different forms in which the same formal logical idea can be expressed. Here are some common English sentences that are all logically equivalent (i.e. they have the same meaning):

- If it is a dog, then it is a mammal.
- All dogs are mammals.
- Every dog is a mammal.
- It is a dog only if it is a mammal.
- Only mammals are dogs.
- It is not a dog unless it is a mammal.
- If it is not a mammal, then it is not a dog.

The final translation is of particular interest. In formal logic terminology, it is called the contrapositive of the original statement.

The Contrapositive

Every formal logic statement has a contrapositive, which is just another way of expressing the same idea. To form the contrapositive, we go through two (or occasionally three) steps:

1. Reverse all of the terms.

2. Negate each term.

3. Swap the *and*s and *or*s in the statement (if applicable).

Let's go through the steps for our original statement: If it is a dog, then it is a mammal.

1. If it is a mammal, then it is a dog.

2. If it is *not* a mammal, then it is *not* a dog.

3. (No *and/or* in the original statement, so skip this step.)

To clarify, we can take a step back from formal logic and just think about what we are saying:

We know that anything that is a dog has to be a mammal. In other words, being a mammal is a requirement of being a dog. So, since all dogs have to be mammals, if we know that something isn't a mammal, then there's no way it could possibly be a dog.

Another way we could have described *if*s and *then*s is as *sufficient clauses* and *necessary clauses*. Being a dog is *sufficient* for being a mammal, since knowing that something is a dog is enough to know that it is also a mammal; being a mammal is *necessary* for being a dog, since knowing that something *isn't* a mammal is enough to know that it *isn't* a dog.

So, to summarize in abstract terms:

Trigger ➔ Result

Lack of the Result ➔ Lack of the Trigger

One other important point to mention — just like in math, double negatives cancel out to become positive. Let's look at a statement that contains a negative and see how this rule affects the contrapositive:

All athletes are *not* fat.

Using our translation table above, we turn this statement into *if-then* form: If an athlete, then not fat.

Following the two steps to form the contrapositive (no need to use step 3 here), we get:

1. If not fat, then an athlete; and

2. If *not* not fat, then *not* an athlete.

Since double negatives cancel out, our final product is: If fat, then not an athlete.

Complex *if-then* Statements

Above we discussed a possible step 3 to forming the contrapositive. Step 3 is used when our original statement doesn't have just one trigger or one result; rather it has multiple triggers or results. Let's look at a few examples:

- If it rains **or** snows, then I will catch a cold.
- If I study hard **and** get a good night's sleep, then I will pass the test.
- If I sleep in, then I will miss my bus **and** be late for work.
- If I go out for dinner, then I will have pasta **or** pizza.

As you can see, complex *if-then* statements include the word *and* or the word *or* and consequently contain multiple triggers or results.

Let's see how we would form the contrapositive of each of these statements:

- If I do *not* catch a cold, then it did *not* rain **and** it did *not* snow.

Notice that we added in step 3 to the process — the *or* in the original became *and* in the contrapositive. Also, notice that we negated each of the three terms. If we had just put *not* in front of the entire second part of the sentence (to get "then it did not rain and snow"), then we would have had a sentence with a different meaning.

So, knowing that I did not catch a cold is sufficient to know that:

1. It did not rain; **and**

2. It did not snow.

- If I do *not* pass the test, then I did *not* study hard **or** I did *not* get a good night's sleep.

What must have happened if I failed the test?

Either:

1. I did not study hard; **or**

2. I did not get a good night's sleep.

This is a good spot for a discussion of one of the major differences between formal logic language and regular English. Normally when we use the word "or", we use what is called an **exclusive** *or*. In other words, if we say "you can have fish or beef for dinner", we mean you can have fish, or beef, **but not both**.

In formal logic, we use what is called an **inclusive** *or*. In other words, there is always an implied "or both" at the end of every *or* statement.

So, there is a third possibility to add to our list of results:

3. I did not study hard **and** I did not get a good night's sleep.

On to our final two examples:

- If I do *not* miss my bus **or** I am *not* late for work, then I did *not* sleep in.

Sleeping in would have definitely led to both missing my bus and being late for work. So, if either of those things did not happen, then there is no possible way that I could have slept in.

- If I do *not* have pasta **and** I do *not* have pizza, then I did *not* go out for dinner.

  We know that if we had gone out for dinner, we definitely would have had either pasta or pizza (or possibly both, if you remember our discussion of *or* above). So, if we do not have pizza and we do not have pasta, then there is no possible way we could have gone out for dinner.

  One phrase that can be tricky to translate is "neither *X* nor *Y*". In formal logic, "neither *X* nor *Y*" translates as "not *X* **and** not *Y*". For example:

- If Bob goes to the party, then neither Fred nor Nancy will go to the party Could

  be translated as:

  If Bob goes, then Fred does not go **and** Nancy does not go. Or, if

  you prefer to symbolize:

  If *B* ➔ not *F* **and** not *N*

Combining *if-then* Statements

In most formal logic puzzles, you are presented with a number of *if-then* statements and your task is to make deductions from them. There are two common ways to combine *if-then* statements to make deductions.

The first type of deduction is formally known as a **deductive syllogism**. Fortunately, we do not need to know the technical terms for the test; we just need to know how to make the deduction. Here is the basic form of the deductive syllogism:

All *A* are *B*;

x is an *A*;

Therefore, *x* is a *B*.

(Remember, "All *A* are *B*" is just another way of saying "If *A*, then *B*".)

To make this abstract concept more concrete, we can revisit our old friends the dogs and the mammals:

All dogs are mammals;

Fido is a dog;

Therefore, Fido is a mammal.

So, the basic elements required to make this kind of deduction are:

1. An *if-then* statement; and

2. An entity (Fido, in our example above) that matches the trigger.

The second type of deduction is formally known as a **hypothetical syllogism**. To make this kind of deduction, we combine multiple *if-then* statements. Here is the basic form:

$A \Rightarrow B$;

$B \Rightarrow C$;

Therefore, $A \Rightarrow C$.

Here is an example:

If I am in Toronto, then I am in Ontario. If I

am in Ontario, then I am in Canada.

Therefore, if I am in Toronto, then I am in Canada.

The basic element required to make this type of deduction is:

1. Two *if-then* statements in which the **result** of one statement is identical to the **trigger** of the second statement.

Putting It All Together

To better illustrate these formal logic concepts, we can tackle a puzzle together. A typical puzzle will have four to six rules and be followed by three to five questions. Here we go:

Alice is throwing a party. Alice will invite at least two boys and at least one girl. The boys that she is considering inviting are Bob, Carlos, Dimitri, Edgar and Frank. The girls that she is considering inviting are Gina, Harriet, Ingrid, Jasmine and Kiku. Alice will adhere to the following guidelines:

There has to be an equal number of girls and boys at the party (including Alice).

If she invites Bob, she has to invite Ingrid.

If she doesn't invite Jasmine, she cannot invite Kiku. She

will invite Dimitri only if she invites Carlos.

She cannot invite Gina unless she invites Frank.

If she invites Frank, then she must invite Kiku and Bob.

The first step is to understand the basic framework of the game. Here, the basic scenario is someone sending out invitations to a party. We have two major numbers restrictions: we need at least two boys and one girl and the number of boys has to equal the number of girls. Numbers rules are often key to solving logic puzzles, so be on the lookout for these rules in both the opening paragraph and the rule set.

Our next step is to make a list of entities and to translate the rules into basic *if-then* format. To avoid confusion later on, it is a good idea to use different symbols for different kinds of entities. For this puzzle, we can use uppercase letters for the boys and lowercase letters for the girls.

Boys: B, C, D, E, F

Girls: g, h, i, j, k

What other girl should we jot down? Why? Alice, of course! We are told that Alice counts as one of the girls for the party, so when we check to make sure that the number of girls is equal to the number of boys we have to remember that Alice is always at the party, no matter who else we choose.

Now we can translate the rules, using our translation guide from earlier in this chapter. It is also an excellent idea to jot down the contrapositive of each rule:

- B ➔ i
  Not i ➔ Not B
- Not j ➔ Not k
  k ➔ j
- D ➔ C
  Not C ➔ Not D
- g ➔ F
  Not F ➔ Not g
- F ➔ k **and** B
  Not k **or** not B ➔ Not F

If you have trouble with any of these translations, then it is worth revisiting the chart above. Learning the most common formal logic translations will make the puzzles go much more smoothly.

At this point we recognize that there are probably a lot of deductions that we can make based on these rules. However, since we are only answering a few questions, it is really not worth checking every possible deduction.
Instead, we are better off making sure that we understand the numbers limitation of each individual rule and then jumping into the questions.

Here is a sample question from this game:

1. If Alice invites Gina, who else does she have to invite to the party?

Whenever we get new information, we want to use it to set off a chain of deductions. We do so by scanning the **triggers** of our *if-then*s. After such a scan, we see that *g* is a trigger in our second-last rule:

g ➔ *F*

So, we immediately know that we need to have Frank at the party. However, our work is not necessarily done — we now need to see if *F* is a trigger in any rules. Sure enough, wespot:

F ➔ *k* **and** *B*

Frank triggers both Kiku and Bob, so we need to add both of them to our invitation list. For now, we are up to *F*, *k*, *g* and *B*. Now we need to check for *k* and *B* as triggers. You should have seen:

B ➔ *i*

and *k*

➔ *j*

So, both Ingrid and Jasmine have to come as well. Once we include these two, we have *B*, *F*, *g*, *i*, *j* and *k* on our list. One more scan for *i* and *j* as triggers shows us that we are finally out of new results.

However, does this mean that we have our final answer? Not necessarily – we have to remember our numbers rule.

So far we have only two boys on our invitation list, B and F, but four girls, g, i, j and k. Also, we have to remember to include our fine hostess, Alice. Since we have five girls attending, we need five boys. Since there are only five boys in total, that means all five of them have to attend! After adding C, D and E to our list, we need to do a quick check of the rules to make sure that we don't have any contradictions (for example, if a rule had said that Dimitri and Edgar never get invited to the same parties, then we would
be in trouble). Adding C, D and E does not lead to any conflicts, so our final guest list is:

Bob, Carlos, Dimitri, Edgar, Frank, gina, ingrid, jasmine, kiku and, of course, alice herself.

Only poor, lonely Harriet gets left off the list.

Ordering Problems

Another type of logic problem you could encounter focuses on how different items are ordered or sequenced in time or space. For example, you could be asked to solve a problem looking at which days of the week certain household activities are scheduled, or at which houses in a row different people live.

Information for these types of problems will not necessarily employ conditional logic clues as discussed above, rather they will have relative position information. For example, Javier will mow the lawn later in the week than the day he completes groceries, or Fiona lives directly to the east of Aliya.

Problems of this type usually benefit from the use of a matrix or table to organize information.

Let’s look at an example:

A teacher is hanging six paintings on the wall in a straight line. The paintings were made by six students: Ahmed, Benjamin, Ciara, Delilah, Eduardo and Faith. The order in which the paintings are hung must conform to the following restrictions:

> Benjamin’s painting must be either first or last.
>
> There must be exactly one painting between Ciara’s and Faith’s.
>
> Ciara’s painting must come after Ahmed’s but before Faith’s.

If Eduardo’s painting is hung fourth, what is a possible arrangement of the paintings? (There may be more than one valid answer; however, you only need to show one.)

As noted above, a matrix would be useful to track which paintings can be hung, or not hung in different positions. We will place the artists initials down the side of the matrix and the positions for the paintings across the top.

	1	2	3	4	5	6
A						
B						
C						
D						
E						
F						

Next, we want to look at each restriction and place that information in the matrix. We will place an ‘X’ in any cell that we know is not a possible match and a ‘✓’ when we confirm a specific requirement. For example, the first clue tells us that Benjamin’s painting must be hung either first or last. From this we can infer that his painting will never be in positions 2 through 5.

	1	2	3	4	5	6
A						
B		X	X	X	X	
C						
D						
E						
F						

The next clue does not yield direct information we can place in our matrix with what we currently know. If there must be exactly one painting between Ciara's and Faith's, we first need information about the position of one of their paintings. For now, let's make a note as a reminder that there is a restriction linking these two paintings.

	1	2	3	4	5	6	
A							
B		X	X	X	X		
C*							*C-?-F or F-?-C
D							
E							
F*							

Since Ciara's painting comes after Ahmed's but before Faith's, we know that Ciara's painting can be neither first nor last, and we can refine our note to eliminate the option of having Faith's painting before Ciara's.

But that is not all. Since we now know that there is exactly one painting separating Ciara's and Faith's, we know Ciara's painting cannot be fifth.

	1	2	3	4	5	6	
A							
B		X	X	X	X		
C*	X				X	X	*C-?-F
D							
E							
F*							

But that is still not all! Valuable information is available for the position of both Ahmed’s and Faith’s paintings. Since Ahmed’s painting must come before Ciara’s; and Ciara’s is followed by one painting then Faith’s, we can infer that Ahmed’s painting must have at lest three paintings after it. Therefore, it cannot be in positions 4, 5, or 6.

	1	2	3	4	5	6	
A				X	X	X	
B		X	X	X	X		
C*	X				X	X	*C-?-F
D							
E							
F*							

As for Faith, we similarly know that three paintings must precede hers, so her painting cannot be in positions 1, 2, or 3.

	1	2	3	4	5	6	
A				X	X	X	
B		X	X	X	X		
C*	X				X	X	*C-?-F
D							
E							
F*	X	X	X				

Now that we have placed all our restrictions, we can look at the question and incorporate that information. We are asked for one possible configuration if Eduardo’s painting is placed fourth.

If Eduardo’s painting is placed fourth, we can infer two useful facts. These are: if Eduardo’s painting is fourth, no other painting can be fourth; and Eduardo’s painting cannot be in the other positions.

	1	2	3	4	5	6	
A				X	X	X	
B		X	X	X	X		
C*	X			X	X	X	*C-?-F
D				X			
E	X	X	X	✓	X	X	
F*	X	X	X	X			

Since we gained new information, it is useful to review if there are any other implications with our other restrictions. Since Faith’s painting can no longer be hung in the fourth position, we know that Ciara’s cannot be in the second, and her painting must be in the third. In turn this means that Faith’s painting must be fifth, with Eduardo’s being the single painting that separates them.

	1	2	3	4	5	6	
A			X	X	X	X	
B		X	X	X	X		
C*	X	X	✓	X	X	X	*C-E-F
D			X	X	X		
E	X	X	X	✓	X	X	
F*	X	X	X	X	✓	X	

We have now established the paintings that must be in positions 3 through 5 based on Eduardo’s being fourth. Since the question asks for a single configuration of the paintings, we are free to select the remaining paintings from the options contained in the matrix.

For example, Ahmed, Delilah, Ciara, Eduardo, Faith, Benjamin is one possible sequence if Eduardo’s painting is fourth that complies with the other restrictions we have been given.

Critical Reasoning Problems

The final type of logic problem you can encounter on the EDT is critical reasoning problem. In these questions you will be asked to select a statement from a list that best reflects the conclusion that can be drawn from a brief statement or paragraph, or an assumption on which the statement or paragraph is based. You must then support your answer.

For example:

Since the turn of the century, many of the brightest computer programmers have sought to start and run successful businesses. However, due to the return on investment demands of their investors, today's programmers must focus on developing products that generate profit. Consequently, computer programming is no longer as creative as it once was.

Which of the following statements best reflects the assumption underlying the preceding argument for the conclusion above to be drawn?

A) All computer programs must lack creativity to be well received.

B) Some computer scientists entirely disregarded creativity and chose instead to pursue profit.

C) A program cannot be both creative and profitable.

D) Computer scientists are obsessed with the profitability of their work.

E) Non-profit institutions use large amounts of software.

The conclusion being drawn by the author of the statement is that 'computer programming is no longer as creative as it once was'.

The argument draws a distinction between creative programming and profitable programming and views these aspects as mutually exclusive.

Therefore, the answer must be C), since if the author believed that it was possible to write both creative and profitable software there would be no basis for the argument that business and profitability drivers have reduced the creativity of the computer programming field.

Pre-Quiz Answer Key

Chapter 1 – Arithmetic

1. $\frac{13}{12}$ or $1\frac{1}{12}$

2. $-\frac{1}{10}$

3. a. 0.75
   b. 0.9
   c. 1.4
   d. 0.625
   e. 6.5
   f. 0.833

4. a. 40
   b. 40
   c. 36
   d. 125
   e. 175
   f. 250
   g. 162
   h. 12
   i. 64

Chapter 2 – Exponents and Radicals

1. 16

2. $x = +6$ or -6

3. a. 27
   b. 39
   c. 72
   d. 412
   e. $3(2^4)$

4. 5

5. 13

6. a. 5
   b. $2\sqrt{5}$
   c. 12
   d. 75
   e. 5
   f. 5

Chapter 3 – Algebra

1. 16

2. $y = -18$

3. a. $x = (\frac{2}{3})y - 5$

 b. $x = (\frac{5}{2})y$

 c. $x = (\frac{4}{3})y - z + \frac{8}{3}$

4. $x = 6$ and $y = -2$

5. $x = 7$ and $y = -\frac{2}{3}$

6. $x = 2$ or $x = -6$

7. $y = 2$ or $y = 10$

8. a. $z^2 + 8z - 48$

 b. $c^2 - 16$

 c. $6x^2 + x - 12$

 d. $xy + 4y - 2x - 8$

 e. $x^3 + 3x^2 - 4x$

Chapter 4 – Co-ordinate Geometry

1.

 a. y-intercept $= -10$; x-intercept $= 5$; slope $= 2$

 b. y-intercept $= 18$; x-intercept $= -2$; slope $= 9$

 c. y-intercept $= 18$; x-intercept $= -9$; slope $= 2$

 d. y-intercept $= 8$; x-intercept $= \frac{16}{3}$; slope $= -\frac{3}{2}$

2. $y = -(\frac{1}{2})x + 5$

3. a. $y = -\left(\frac{1}{3}\right)x + \frac{1}{3}$

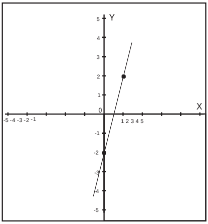
 b. $y = 3x - 4$

 c. $y = x + 1$

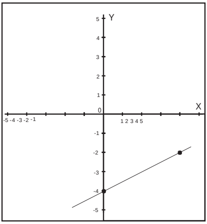
 d. $y = -3x + 2$

4. 5

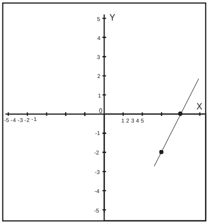
5a



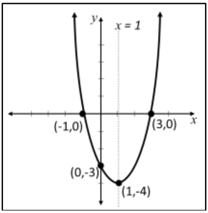
5b



5c



5d



Chapter 5 – Sequences and Summations

1. $S_1 = 6$
   a. $S_2 = 8$
   b. $S_3 = 12$
   c. $S_4 = 20$

2. $R4 = -16$
   a. $R_3 = -3$
   b. $R_2 = 2$
   c. $R_1 = 9$

3. $Q_1 + Q_2 + Q_3 + Q_4 = 6 + 3 - 6 + 75 = 78$

4. $(2^2 - 3(2) + 7) + (3^2 - 3(3) + 7) + (4^2 - 3(4) + 7) + (5^2 - 3(5) + 7)$
   $= (4 - 6 + 7) + (9 - 9 + 7) + (16 - 12 + 7) + (25 - 15 + 7)$
   $= 5 + 7 + 11 + 17 = 40$

Chapter 6 – Logarithms

1. a. $x = 2$
   b. $x = 4$
   c. $x = 5$
   d. $x = 2$
   e. $x = 216$
   f. $x = 64$
   g. $x = 2$
   h. $x = 100000$

2. a. $\log_2 5$
   b. $\log 100000 = 5$
   c. $\log_3 27 = 3$
   d. $\log 10 = 1$

Chapter 7 – Logic

1. a. If a person is in Toronto, then he or she is in Ontario
   b. If Bob wins the race, then Sheila does not enter.
   c. If Mary leaves the company, then Sam will get the promotion.
   d. If Yuko goes to a movie, then Philippe does not go.
   e. If Sabrina goes to a movie, then Juan also goes; if Juan goes to a movie, then Sabrina also goes.
   f. If Hassan is not on vacation, then Dietrich is on vacation.
   g. If it rains or snows, then I get a headache.
2. a. If a person is not in Ontario, then he or she is not in Toronto.
   b. If Sheila enters the race, then Bob does not win.
   c. If Sam does not get the promotion, then Mary does not leave the company.
   d. If Philippe goes to a movie, then Yuko does not go.
   e. If Juan does not go to a movie, then Sabrina does not go; if Sabrina does not go to a movie, then Juan does not go.
   f. If Dietrich is not on vacation, then Hassan is on vacation.
   g. If I do not get a headache, then it did not rain and it did not snow.
3. 1. d. She doesn't buy the desk. 2.
   c. 4