References:

Essential Inputs:

- Cournot-Nash duopoly;
- Stackelberg Leadership model

Scenario

- an established firm (incumbent) and an entrant;
- earlier treatment of the issue (Bain-Sylos): entrant believes that the incumbent will maintain its output at pre-entry level \(\Rightarrow\) the incumbent is a conventional Stackelberg leader:

BUT

1. faced with entry with certainty, the incumbent will try to make an accommodating change in its output;
2. the incumbent would like to threaten to respond to entry with a predatory increase in its output but the issue is one of the credibility of the threat.

Comment
1. An incumbent can logically only threaten to do what it will actually do in the face of entry. This is common knowledge when there is complete information. This may change if the entrant faces some uncertainty. For example, the entrant may not know the technology of the incumbent and therefore the entrant may be uncertain of the optimal response of the incumbent to entry. Still there will be expectations. This is absent from this setting. The basic point of the paper is to work out what the options are for the incumbent and what constitutes credible response to proposed entry.

2. We can define three qualitative responses by the incumbent and two easily observable equivalents for entrants. Start with the observables for entrants: suppose that we are aware of potential entry with certainty: either entry occurs or it does not. The qualitative responses of the incumbent to proposed entry are

- play monopoly output and entry does not occur: entry is blockaded;
- entry cannot be prevented and so is accommodated and incumbent picks best response to entry;
- entry could be prevented: firm chooses to prevent entry or to accommodate entry according to which strategy maximizes profits of incumbent.

Assumptions

- 2 firms: \( i = 1, 2 \);
- fixed costs denoted by \( f_i \);
- fixed coefficients production technology (Leontief technology). We will measure inputs in terms of their associated output capacities, i.e., in term of output units. Corresponding costs for inputs are therefore: (i) capital costs per unit of output are denoted by \( r_i \), and (ii) corresponding labor costs per unit of output are denoted by \( w_i \);
- incumbent firm, firm 1, installs capacity \( k_1 \); this can be increased but not decreased, i.e., this is sunk capital;
- firm 2, the entrant, observes this choice and either enters or not;

No production occurs prior to firm 2’s decision but a Nash equilibrium is established immediately after.

Use the material to consider the profit maximizing decisions of the incumbent once the incumbent has installed capacity. Once we understand this, we can define the profit maximizing capacity decision for the incumbent. Suppose that firm 1 establishes capacity denoted by \( \tilde{k}_1 \).

Costs:
Then with capacity in place, cost functions are different when the firm wishes to produce output below the capacity limit and when it wishes to produce output above the capacity limit. In particular,

\[ C_1 = f_1 + r_1 \bar{k}_1 + w_1 x_1 \text{ for } x_1 \leq \bar{k}_1 \]
\[ C_1 = f_1 + (r_1 + w_1) x_1 \text{ for } x_1 > \bar{k}_1 \]

This gives the corresponding marginal cost relationships as follows:

\[ MC_1 = w_1 \text{ for } x_1 \leq \bar{k}_1 \]

and

\[ MC_1 = (w_1 + r_1) \text{ for } x_1 > \bar{k}_1 \]

Firm 2 has no prior commitment in capacity so that its cost function is

\[ C_2 = f_2 + (r_2 + w_2) x_2 \]

**Revenues:**

Revenues are defined by

\[ R^i(x_1, x_2) \text{ with } R^i_{ij} > 0 \text{ and } R^i_{ii} < 0, \ R^i_{ij} < 0 \]

The sign \( R^i_{ij} < 0 \) is important for the result that firm 1 never holds excess capacity. We shall develop this from BGK once we have presented the Dixit model. Note that \( R^i_{ij} < 0 \) means that the two outputs are strategic substitutes; \( (R^i_{ij} > 0 \) would mean that the two outputs are strategic complements.)

**Game:**

The game is a sequential move game: firm 1 moves first with knowledge of the nature of the post-entry Nash equilibrium. The strategic problem facing firm 1 is to decide on its investment decision knowing the response of firm 2 in the post-entry game. We will use a reaction path diagram to solve this problem.

We need to map the technology of firm 1 into a reaction type diagram. Go to Figure 1. Note that the jumps in the top diagram are relevant for the reaction path discontinuities in the bottom diagram. That is, develop the reaction paths for firm 1. The sole purpose of this is to set out the limits for the strategies of firm 1, the incumbent firm. Remember that \( MR_1 \) is decreasing in \( x_2 \).

The FOC for firm 1 are as follows:

\[ R^1_1 = w_1 \text{ for } x_1 \leq \bar{k}_1 \]

and

\[ R^1_1 = (w_1 + r_1) \text{ for } x_1 > \bar{k}_1 \]
The slopes of both reaction paths for firm 1 are the same and given by

$$\frac{dx_2}{dx_1} = -\frac{R_{11}}{R_{12}} < 0$$

provided outputs are strategic substitutes.

For notational ease, I follow Dixit and if a point in \((x_1, x_2)\) space is denoted by \(J\), the corresponding output choices will be \((J_1, J_2)\) where \(J_i\) is the output of the \(i^{th}\) firm. From Figure 1, \(M'M'\) and \(N'N'\) have special meaning. The relevant end points are \(M = (M_1, 0)\) and \(N = (N_1, 0)\). By definition \(M_1\) and \(N_1\) are the output choices of 1 when the output of 2 equals 0. In particular, \(M_1\) is the capacity choice ‘when expansion costs matter’; \(N_1\) is relevant when there is sufficient capacity installed and only variable costs matter. \(M_1\) is the monopoly output level and is given by

\[(w_1 + r_1) = R_1^1(x_1, 0)\]

\(N_1\), when only variable costs matter is given by

\[w_1 = R_1^1(x_1, 0)\]

These points are illustrated in Figure 2.

Firm 2 has no prior commitment and its reaction function is straightforward.

The advantage to 1 is that it can chose \(\bar{k}_1\) in advance and so has the privilege of determining which reaction function it will present to the post-entry game. Go to Figure 3.

Suppose that 2’s reaction function intersects at \(T = (T_1, T_2)\) and \(V = (V_1, V_2)\) as shown. The points \(T\) and \(V\) may interpreted as ‘extreme’ Nash equilibrium.

Identify Some Relevant Points on the Diagram:

1. \(M_1\) is the monopoly capacity level;
2. firm 1 will never install capacity less than \(T_1\), i.e. \(T_1\) is a Nash equilibrium.
3. when the capacity installed is sufficiently large so that capacity is non-binding, then we will be at \(V_1\): capacity threats larger than \(V_1\) are not credible; in particular capacity larger than \(N_1\) is never credible for even if firm 2 were not present, capacity larger than \(N_1\) would not be profit maximizing (or loss minimizing).

All of this means that the relevant range for 1 to maximize against 2’s output choice is given by the segment \(TV\); that is, this is the relevant segment for 1 to manipulate the initial conditions of the post-entry game. Now we come to 1’s ex ante decision: what is the pre-entry choice of firm 1?

Pre-entry

At all points of observable output, the established firm produces where \(x_1 = \bar{k}_1\) — as I mentioned above this has to do with the strategic substitute nature of the two outputs.
We will shortly see how this is altered absent this condition. With no excess capacity, the corresponding profit function for firm 1 is

\[ \pi^1 = R^1(x_1, x_2) - (w_i + r_i)x_i - f_i \]

We assume that \( \pi^1 > 0 \) but there are various cases for \( \pi^2 \). In fact, there are 3 cases as I outlined in my introductory remarks. Two of these cases are ‘limiting’ cases and the third case is a more interesting ‘interior’ case.

**Cases (Classification of Outcomes)**

1. \( \pi^2(T) < 0 \): no entry and firm 1 acts as a monopolist setting \( x_1 = M_1 \).

2. \( \pi^2(V) > 0 \): 1 cannot prevent entry and therefore 1 looks for the best duopoly situation. This means that firm 1 is a Stackelberg firm (\( \max_{x_1} \pi^1(x_1, x_2) \)) given the reaction path of firm 2 \( (x_2 = x_2(x_1)) \). The relevant diagram is given by Figure 4.

3. \( \pi^2(T) > 0 > \pi^2(V) \): Then \( \exists \) a point \( B \) along TV s.t. \( \pi^2(B) = 0 \) where \( B = (B_1, B_2) \). Thus an incumbent setting \( k_1 > B_1 \) prevents entry. There are a number of sub cases to consider. All of these are illustrated on the final diagram:
   (a) \( B_1 < M_1 \): then the incumbent’s monopoly choice is entry blockading;,
   (b) \( B_1 > M_1 \): then the incumbent *could* bar entry but could do so only by maintaining capacity at a level greater than it would want as a monopolist. If entry is not barred then we label the Stackelberg solution by \( S = (S_1, S_2) \). This leads to two further sub cases. These are easy to state and can be illustrated on the final diagram.
      i. \( \pi^1(S) < \pi^1(B_1, 0) \) \( \implies \) choose \( B_1 > M_1 \) and entry is prevented;
      ii. \( \pi^1(S) > \pi^1(B_1, 0) \) \( \implies \) choose \( S_1 \) and entry is accommodated; there is a duopoly at \( S \); \( S \) is a post-entry Nash equilibrium.

Go to Figure 5.

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**Question:** Will firm 1 ever install capacity that is in the end unused?

**Logic:** The logic to the response is that firm 1 will never install capacity unused in any circumstance. It will not install capacity that is not needed if entry is certain. Firm 1 might install capacity that is unused if there is no entry but the capacity serves the purpose of deterring entry in the first place. By the definition of credibility of the threat, this must be capacity that would be used in the duopolistic post-entry Nash equilibrium. Suppose that the reaction paths look like those in Figure 5.

What is critical is that firm 1’s reaction curves turn inwards near to the \( x_2 = 0 \) axis. For firm 1 to hold idle capacity, firm 1 must wish to produce more given its marginal costs for
some \( x_2 > 0 \) than for \( x_2 = 0 \). But Dixit’s analysis always assumes that the reaction path has a negative slope, or equivalently \( R_{ij}^1 < 0 \). That is, that \( x_1 \) and \( x_2 \) are always strategic substitutes. But as the above diagram shows, if these outputs are strategic complements over the appropriate range, then idle capacity may be observed when the job of the excess capacity is to deter entry but once this is accomplished some of the installed capacity is not used by the incumbent.

Based on a numerical example using a constant elasticity demand curve (of the type \( P = c(x_1 + x_2)^{1/\eta} \)), BGK show that if firm 1 installs a capacity of \( N_1' \), the equilibrium is at \((M_1,0)\), i.e., entry is deterred and some of the installed capacity is not utilized.
When $X_1 = X_2$ and strategy substitutes, $H_1$ shift to right as $X_2$ falls.
Figure 5

$N^1 = (N^1_1, N^1_2)$

$N = (N_1, 0)$  $(N_1^1, 0) X_1$